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Equilibrium Polarization Formula for Electron Storage Rings Including Transverse Recoils in All Planes

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Abstract

The extension of the Derbenev-Kondratenko formula for radiative electron polarization in a high-energy storage ring, to include the effects of electron recoils in all transverse planes, is presented.

Electrons in a high-energy storage ring become polarized by the emission of synchrotron radiation, and a formula for the equilibrium degree of polarization was given by Derbenev and Kondratenko.[1] This formula contains terms which describe the longitudinal recoil of an electron due to a photon emission. In a recent paper,[2] Barber and the author extended the Derbenev-Kondratenko formula to take into account the effects of vertical recoils (i.e. recoils in the direction of the local magnetic field) as well as the longitudinal recoils (Eq. (42) in Ref. 2). Here the full formula, with transverse recoils in all planes, is presented. The formalism used follows that of Refs. 1 and 2.

It is convenient to use a right-handed set of unit vectors $\{\hat{i},\hat{j},\hat{k}\}$, with $\hat{i}$ along the outward normal to the trajectory, $\hat{j}$ along the tangent, and $\hat{k} = \hat{b}$, the direction of the local magnetic field. The electron spin quantization axis is called $\hat{n}$ (see Ref. 2 for details).

Then the change in $\hat{n}$ due to a photon emission is given by the derivatives

$$\vec{d} = \gamma \frac{\partial \hat{n}}{\partial \gamma} , \qquad \vec{e} = \frac{\partial \hat{n}}{\partial \beta_i} , \qquad \vec{f} = \frac{\partial \hat{n}}{\partial \beta_k} , \tag{1}$$

where $\vec{\beta} = \vec{v}/c$ and γ is the electron energy in units of rest energy. The above derivatives come from the operator (see Ref. 2)

$$\frac{\partial}{\partial \vec{p}} = \frac{1}{\gamma mc} \left[\hat{j} \gamma \frac{\partial}{\partial \gamma} + \hat{i} \frac{\partial}{\partial \beta_i} + \hat{k} \frac{\partial}{\partial \beta_k} \right] , \tag{2}$$

where $\vec{p}$ is the electron momentum. The vector $\vec{d}$ arises from the longitudinal recoil of an electron while the vectors $\vec{e}$ and $\vec{f}$ arise from transverse recoils along $\hat{i}$ and $\hat{k}$, respectively. Algorithms to calculate $\hat{n}$ and $\vec{d}$ have been published in Refs. 3 and 4, and can easily be extended to calculate $\vec{e}$ and $\vec{f}$ too. The polarization formula is

$$P = \frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho|^3} \left\{ \hat{b}.\hat{n} - \hat{b}.\vec{d} - \frac{1}{3\gamma} \hat{v}.\vec{f} + \frac{1}{3\gamma} \hat{n}.(\vec{d} \times \vec{e}) \right\} \right\rangle}{\left\langle \frac{1}{|\rho|^3} \left\{ 1 - \frac{2}{9}(\hat{n}.\hat{v})^2 + \frac{11}{18}|\vec{d}|^2 + \frac{13}{90\gamma^2}(|\vec{e}|^2 + |\vec{f}|^2) - \frac{1}{9\gamma} \frac{\dot{\vec{v}}}{|\vec{v}|}.(\hat{n} \times \vec{f}) \right\} \right\rangle} . \tag{3}$$

Here ρ is the local radius of curvature of an electron trajectory, and the angular brackets denote an average over the accelerator azimuth and distribution of particle orbits.

Let us conclude by recalling the conditions under which the above formula is derived. Eq. (3) is calculated by treating the electron orbital and spin motion to the leading order in $\hbar$. However, it includes all orders of spin resonances, and arbitrary nonlinear (but integrable) orbital dynamics in the accelerator. The accelerator geometry is arbitrary, and the effects of magnet fringe fields, and magnet misalignments, are also included. It is assumed that synchrotron radiation is the dominant mechanism of causing spin-flip, and so beam-beam interaction is not included. It is also assumed that the polarization buildup time is much longer than the orbital radiation damping time, which is a very good approximation in all modern storage rings. The direction of the polarization vector is given by $\langle \hat{n} \rangle$, where the angular brackets denote an ensemble average over the orbital action-angle variables. In order to evaluate the above formula for a practical accelerator model,

it may be necessary to make further approximations, but such approximations pertain to the *algorithms*, not to the polarization formula itself.

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